Transmitted Powers of Waves through Superconductor-Dielectric Photonic Crystal

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***Abstract*—We Consider a periodic superconductor-dielectric photonic crystal situated between two half free spaces. A polarized plane wave is obliquely incident on it. The reflection and transmission of electromagnetic waves by the crystal are analyzed theoretically and numerically with the emphasis on the penetration depth of the superconductor. Maxwell's equations are used to determine the electric and magnetic fields of the incident waves at each layer. Snell's law is applied and the boundary conditions are imposed at each layer interface to calculate the reflected and transmitted coefficients of the structure. The reflected and transmitted powers of the crystal are determined using these coefficients by a recursive method. In the numerical results, the mentioned powers are computed and illustrated as versus the wavelength, angle of incidence and dielectric thickness.**

***Index Terms*—Electromagnetic waves, penetration depth, photonic crystal, reflection, transmission, superconductor**

I. Introduction

Photonic crystals are structures with periodically modulated dielectric constants that affect the propagation of electromagnetic waves in the same way as the periodic potential in a semiconductor crystal affects the electron motion by defining allowed and forbidden electronic energy bands. Photonic crystals have been studied in one form or another before the term photonic crystal had been known. In 1972, V. P. Bykov [1] has studied spontaneous emission in a periodic structure. In 1975, V. P. Bykov [2] has investigated spontaneous emission from a medium with band structure. K. Ohtaka [3] has studied the energy band of photons and low energy photon diffraction in 1979. The term "photonic crystal" was first used by Eli Yablonovitch and Sajeev Jojn published two milestone papers on photonic crystals in 1987 [4], [5]. They have generated spectral regions named photonic band gaps where light can not propagate in a manner analogous to the formation of electronic band gaps in semiconductors. After 1987, the number of research papers concerning photonic crystals began to grow theoretically and experimentally. By 1991, E. Yablonovitch [6] has demonstrated the first three-dimensional photonic band-gap in the microwave regime. In 1996, T. Karauss [7] has made the first demonstration of a two-dimensional photonic crystal at optical wavelengths. P. Ordejon [8] has analyzed theoretically n-order tight binding methods for electronic structure and molecular dynamics in 1998. In 2000, A. Blanco et al. [9] have performed large-scale synthesis of a silicon photonic crystal with a complete three-dimensional band gap near 1.5 micrometers. Z. Sun et al. [10] have analyzed the optical transmission through a nanoslit collection structure shaped on a metal layer with thin film thickness. M. Ricci et al. [11] have investigated the dielectric losses in a superconducting photonic crystal. Arafa et al. [12], [13] have studied superconducting photonic crystals at microwave, millimeterwaves and far-infrared frequencies. This paper is interested in transmission and reflection of electromagnetic waves by a superconductor/dielectric photonic crystal consisting of N periods. We consider the photonic crystal is embedded in vacuum and a monochromatic plane electromagnetic wave is obliquely incident on it. The electric and magnetic fields are determined in each region using Maxwell's equations. Then Snell's law is applied and the boundary conditions are imposed at each interface to obtain the reflection and transmission coefficients. The reflected and transmitted powers of the structure are presented in terms of these coefficients. In the numerical analysis a recursive method [14], [15] is used to calculate the mentioned powers as a function of wavelength, angle of incidence and the slab thickness when the penetration depth of the superconductor changes. To check the results of the used analysis in these calculations, the conservation law of energy given in [16], [17] is checked and it is clear that it is satisfied for all examples.

II. Theory

Consider a pair of superconductor (ε_2, μ_0) and dielectric (ε_3, μ_0) materials is situated between two half free spaces (ε_0, μ_0). A perpendicular polarized plane wave in region 1 is incident on the plane z = 0 at some angle θ relative to the normal to the boundary (see Fig. 1).

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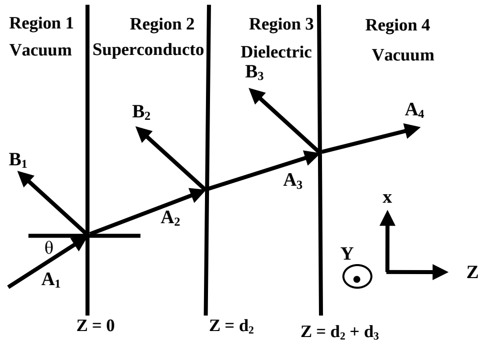


Figure 1. The proposed waveguide structure.

The electric field in each region is [18], [19]:

$$\vec{E}_\ell = \left(A_\ell e^{ik_{\ell z}z} + B_\ell e^{-ik_{\ell z}z}\right)e^{i(k_{\ell x}x-\omega t)}\,\hat{y} \tag{1}$$

To find the corresponding magnetic field $\vec{H}_\ell$, we start with Maxwell's equation $\vec{\nabla}\times\vec{E}_\ell = -\partial\vec{B}/\partial t$, substituting $\vec{B}=\mu_\ell\vec{H}_\ell$ and solving for $\vec{H}_\ell$ yield:

$$\vec{H}_\ell = \frac{1}{\mu_\ell\omega}\left[\begin{array}{l}(A_\ell k_{\ell x}e^{ik_{\ell z}z} + B_\ell k_{\ell x}e^{-ik_{\ell z}z})\hat{z} + \\ (-A_\ell k_{\ell z}e^{ik_{\ell z}z} + B_\ell k_{\ell z}e^{-ik_{\ell z}z})\hat{x}\end{array}\right]e^{i(k_{\ell x}x-\omega t)} \tag{2}$$

Where A_ℓ and B_ℓ are the amplitude of forward and backward traveling waves (ℓ = 1, 2, 3, 4), $k_\ell = n_\ell\,\omega/c$ is the wave vector inside the material and n_ℓ is the refractive index of it.

Matching the boundary conditions for $\vec{E}$ and $\vec{H}$ fields at each layer interface, that is at z = 0, $E_{1y}=E_{2y}$ and $H_{1x}=H_{2x}$ and so on. This yields six equations with six unknown parameters [18]:

$$A_1 + B_1 = A_2 + B_2 \tag{3}$$

$$\frac{k_{1z}}{\mu_1}\left(A_1 - B_1\right) = \frac{k_{2z}}{\mu_2}\left(A_2 - B_2\right) \tag{4}$$

$$A_2e^{ik_{2z}d_2} + B_2e^{-ik_{2z}d_2} = A_3e^{ik_{3z}d_2} + B_3e^{-ik_{3z}d_2} \tag{5}$$

$$\frac{k_{2z}}{\mu_2}\left(A_2e^{ik_{2z}d_2} - B_2e^{-ik_{2z}d_2}\right) = \frac{k_{3z}}{\mu_3}\left(A_3e^{ik_{3z}d_2} - B_3e^{-ik_{3z}d_2}\right) \tag{6}$$

$$A_3e^{ik_{3z}(d_2+d_3)} + B_3e^{-ik_{3z}(d_2+d_3)} = A_4e^{ik_{4z}(d_2+d_3)} \tag{7}$$

$$\frac{k_{3z}}{\mu_3}\left(A_3e^{ik_{3z}(d_2+d_3)} - B_3e^{-ik_{3z}(d_2+d_3)}\right) = \frac{k_{4z}}{\mu_4}A_4e^{ik_{4z}(d_2+d_3)} \tag{8}$$

Where $k_{1x}=k_{2x}=k_{3x}=k_{4x}\equiv$ Snell's law and:

$$k_{\ell z} = \frac{\omega}{c}\sqrt{n_\ell^{\,2} - n_1^{\,2}\sin^2\theta} \tag{9}$$

Fresnel coefficients (interface reflection and transmission coefficients r, t respectively) for perpendicular polarized light are given by [20]:

$$r_{ij} = \frac{\mu_j k_{iz} - \mu_i k_{jz}}{\mu_j k_{iz} + \mu_i k_{jz}} \tag{10}$$

$$t_{ij} = \frac{2\mu_j k_{iz}}{\mu_j k_{iz} + \mu_i k_{jz}} \tag{11}$$

Where i, j correspond to any two adjacent media.

The reflection and transmission coefficients R and T respectively of the structure are found by [15], [21]:

$$R = \frac{B_1}{A_1} = \frac{r_{12} + r_{12}r_{23}r_{34}e^{i2k_{3z}d_3} + r_{23}e^{i2k_{2z}d_2} + r_{34}e^{i2(k_{2z}d_2+k_{3z}d_3)}}{1 + r_{23}r_{34}e^{i2k_{3z}d_3} + r_{12}r_{23}e^{i2k_{2z}d_2} + r_{12}r_{34}e^{i2(k_{2z}d_2+k_{3z}d_3)}} \tag{12}$$

$$T = \frac{A_4}{A_1} = \frac{t_{12}t_{23}t_{34}e^{i(k_{2z}d_2+k_{3z}d_3)}}{1 + r_{23}r_{34}e^{i2k_{3z}d_3} + r_{12}r_{23}e^{i2k_{2z}d_2} + r_{12}r_{34}e^{i2(k_{2z}d_2+k_{3z}d_3)}} \tag{13}$$

The reflectance R' and transmittance T' of the structure are given by:

$$R' = RR^*,\; T' = \frac{k_{4z}}{k_{1z}}TT^* \tag{14}$$

Where R^* and T^* are the complex conjugate of R and T respectively. The law of conservation of energy is given by [5]:

$$R' + T' = 1 \tag{15}$$

For n-layers structure shown in Fig. 2 , R and T are calculated as follows [14], [15]:

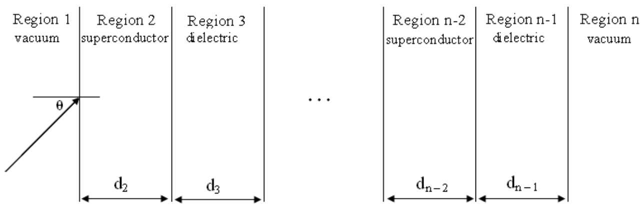


Figure 2. Oblique incidence of electromagnetic waves on superconductor/dielectric photonic crystal embedded in vacuum.

$$R_n = r_{n-1,n} \tag{16}$$

$$R_{n-1} = \frac{r_{n-2,n-1} + R_n e^{i2k_{(n-1)z}d_{n-1}}}{1 + r_{n-2,n-1}R_n e^{i2k_{(n-1)z}d_{n-1}}} \tag{17}$$

$$R_{n-2} = \frac{r_{n-3,n-2} + R_{n-1} e^{i2k_{(n-2)z}d_{n-2}}}{1 + r_{n-3,n-2}R_{n-1} e^{i2k_{(n-2)z}d_{n-2}}} \tag{18}$$

:
:

Continue on the same procedure until R_2 is reached which is the reflectance of the structure as a whole.

$$R_2 = \frac{r_{12} + R_3 e^{i2k_{2z}d_2}}{1 + r_{12} R_3 e^{i2k_{2z}d_2}} \quad (19)$$

The same procedure is performed for T_2:

$$T_n = t_{n-1,n} \quad (20)$$

$$T_{n-1} = \frac{t_{n-2,n-1} T_n e^{i2k_{(n-1)z}d_{n-1}}}{1 + r_{n-2,n-1} R_n e^{i2k_{(n-1)z}d_{n-1}}} \quad (21)$$

$$T_{n-2} = \frac{t_{n-3,n-2} T_{n-1} e^{i2k_{(n-2)z}d_{n-2}}}{1 + r_{n-3,n-2} R_{n-1} e^{i2k_{(n-2)z}d_{n-2}}} \quad (22)$$

:
:

$$T_2 = \frac{t_{12} T_3 e^{i2k_{2z}d_2}}{1 + r_{12} R_3 e^{i2k_{2z}d_2}} \quad (23)$$

Where d_2, d_{n-1} and d_{n-2} are thicknesses of layers 2, n-1and n-2 respectively.

For the superconductor in regions 2, 4, 6, … , an electrodynamic behavior is modeled using the two-fluid theory [22], [23], [24]. The complex conductivity σ can be expressed as:

$$\sigma = \sigma_1 - i\sigma_2 \quad (24)$$

Where the real part indicates the loss contributed by normal electrons, and the imaginary part is due to super electrons, expressed as [24], [25] $\sigma_2 = 1/\omega\mu_0\lambda_\ell^2$, where λ_ℓ is the London penetration depth which depends on the absolute temperature. We shall consider the loss-less case of superconductor, meaning that the real part of the complex conductivity of the superconductor can be neglected and consequently it becomes $\sigma = -i\sigma_2 = -i(1/\omega\mu_0\lambda_\ell^2)$. The relative permittivity of the superconductor can be obtained by [24]:

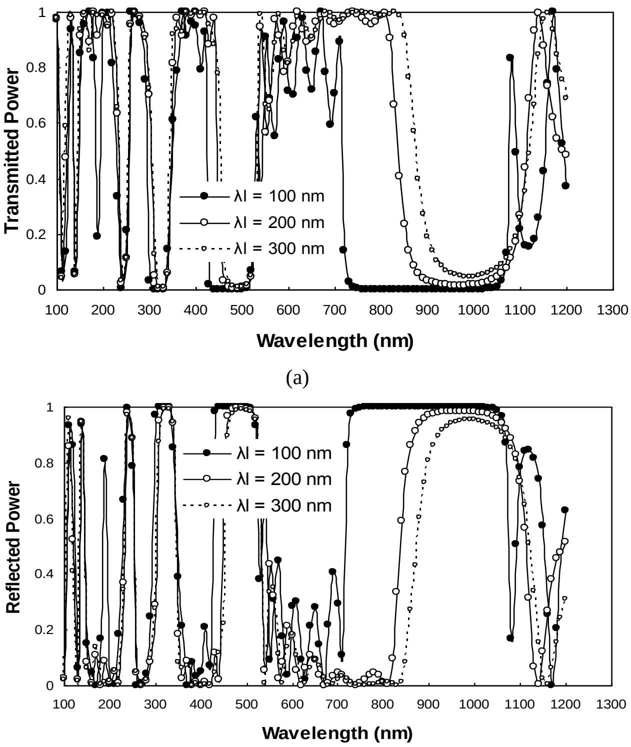
$$\varepsilon_{r2} = 1 - c^2/\omega^2\lambda_\ell^2 \quad (25)$$

Where c is the velocity of light.

III. Numerical Results And Applications

In this section, the transmitted and reflected powers of the photonic crystal described in Fig. 2 are calculated numerically as a function of wavelength, angle of incidence and dielectric thickness for changing values of the penetration depth. We have used the superconductor described in "(25)" and Fluorite (CaF_2) of refractive index 1.434 as a dielectric in each period. The central wavelength is assumed to be λ_0 = 600 nm, the thickness of each period is $2\lambda_0/3$ (thickness of superconductor is 100 nm and that of Fluorite is 300 nm) and the number of periods N = 7. Three values of the penetration depth are considered, λ_ℓ = 100, 200, 300 nm. All regions are assumed to be loss-less and the permeabilities of them are equal to the permeability of free space, $\mu_\ell = \mu_0$.

Fig. 3 shows the transmitted and reflected powers as a function of the wavelength at the incidence angle of 30° when the penetration depth changes. The wavelength is changed between 100 nm and 1200 nm, this range includes ultraviolet, visible and near infrared. We can see that, the transmitted and reflected powers have an oscillation behavior in the range 100-700 nm (for λ_ℓ = 100 nm), 100-900 nm (for λ_ℓ = 200 nm) and 100-950 (for λ_ℓ = 300), this range increases with λ_ℓ. In the ranges 700-1050 nm (for λ_ℓ = 100 nm), 850-1100 nm (for λ_ℓ = 200 nm) and 950-1050 nm (for λ_ℓ = 300 nm), the transmitted power is very small while the reflected power is very large. The wide of these ranges decreases with λ_ℓ. Moreover, in these ranges the transmitted power increases while the reflected power decreases with λ_ℓ.

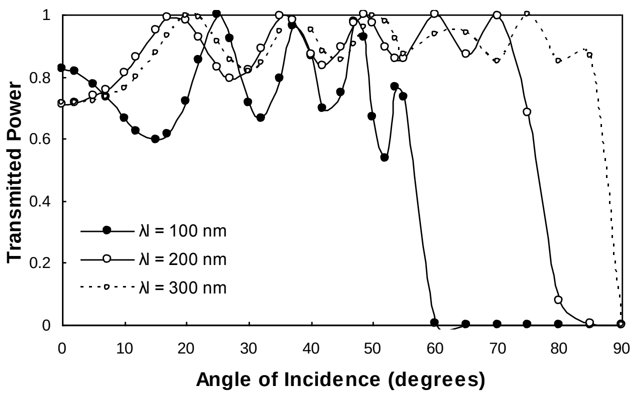


(a)

(b)

Figure 3. The transmitted and reflected powers as a function of wavelength.

Fig. 4 presents the transmitted and reflected powers versus the angle of incidence for 600 nm wavelength. The angle of incidence is changed between 0° and 90° to realize all possible angles of incidence. It can be seen that, the transmitted and reflected powers show increasing, decreasing and oscillating behaviors in different ranges of the angle of incidence. These ranges starts 0° to 60° (for λ_ℓ = 100 nm), from 0° to 80° (for λ_ℓ = 200 nm) and from 0° to 90° (for λ_ℓ = 300 nm). After these ranges the transmitted power is minimum (≈ 0) while the reflected power is maximum (≈ 1).



(a)

Figure 4. The transmitted and reflected powers versus the angle of incidence.

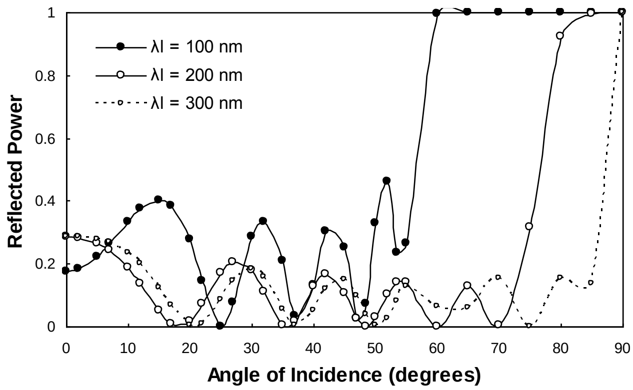


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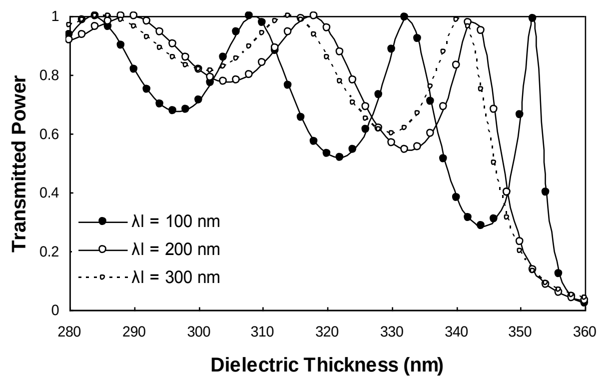
Figure 4. (continue) The transmitted and reflected powers versus the angle of incidence.

Fig. 5 illustrates the transmitted and reflected powers versus the dielectric thickness at the incident angle of 30°. The dielectric thickness is changed from 280 nm to 360 nm. As it is confirmed from the figure, both of the transmitted and reflected powers change periodically with thickness. The maximum values for the transmitted power are the same for all values of λ_ℓ, while the minimum values of it increase with λ_ℓ. For the reflected power, the minimum values are the same while the maximum values of it increases with λ_ℓ. The degree of ripples increases with thickness for all values of λ_ℓ in the case of transmitted and reflected powers.



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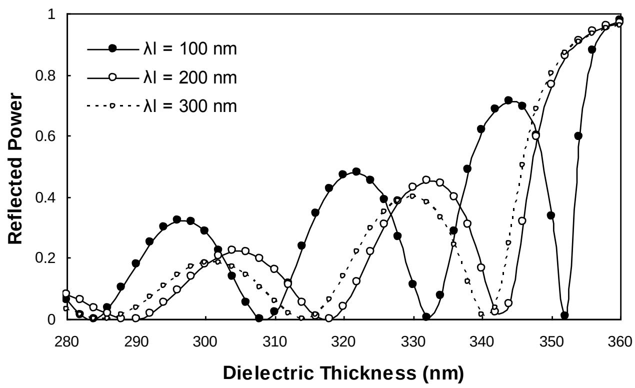

(b)

Figure 5. The transmitted and reflected powers against dielectric thickness.

IV. Conclusion

In this paper, the reflection and transmission characteristics of the electromagnetic radiation propagation through a superconductor/dielectric (CaF_2) photonic crystal are studied in detail with the effect of the penetration depth. The required equations for the electric and magnetic fields in each region are derived by Maxwell's equations. Then Snell's law is applied and the boundary conditions are imposed to calculate the reflection and transmission coefficients of the crystal. Finally, the reflected and transmitted powers as a function of wavelength, angle of incidence and the dielectric thickness are investigated numerically to observe the effect of thee penetration depth on them. As it can be seen from the theoretical and the numerical results, if the penetration depth changes, the characteristic of the powers will be affected by this change. Numerical examples are already presented to illustrate the paper idea and to prove the validity of the obtained results. Moreover the law of conservation of energy is satisfied throughout the performed computations for all examples.

The results obtained in this paper can be helpful to design new devices, apparatus, components at the millimeter wave, optical, and microwave regimes. Furthermore, these results open a way to think how the availability of the penetration depth will change the functionality of a device with superconductor/dielectric photonic crystal.

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